Heat and Mass Transfer: Fundamentals & Applications
Fourth Edition
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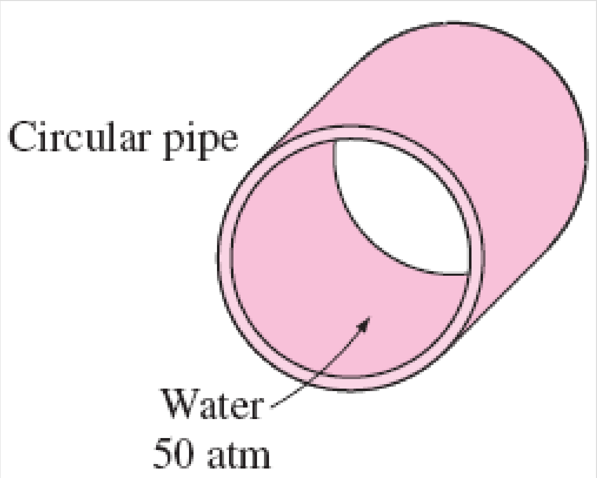
Chapter 8
INTERNAL FORCED CONVECTION

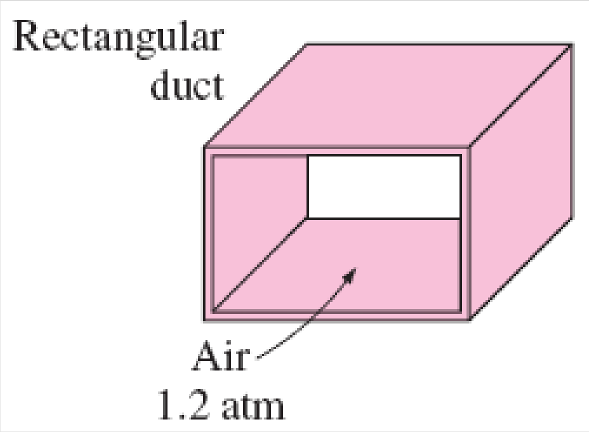
Objectives

- Obtain average velocity from a knowledge of velocity profile, and average temperature from a knowledge of temperature profile in internal flow
- Have a visual understanding of different flow regions in internal flow, and calculate hydrodynamic and thermal entry lengths
- Analyze heating and cooling of a fluid flowing in a tube under constant surface temperature and constant surface heat flux conditions, and work with the logarithmic mean temperature difference
- Obtain analytic relations for the velocity profile, pressure drop, friction factor, and Nusselt number in fully developed laminar flow
- Determine the friction factor and Nusselt number in fully developed turbulent flow using empirical relations, and calculate the heat transfer rate

INTRODUCTION

- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- Although the theory of fluid flow is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.
- Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.





For a fixed surface area, the circular tube gives the most heat transfer for the least pressure drop.

Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

The fluid velocity in a pipe changes from *zero* at the wall because of the no-slip condition to a maximum at the pipe center.

In fluid flow, it is convenient to work with an *average* velocity V_{avg}, which remains constant in incompressible flow when the cross-sectional area of the pipe is constant.

The average velocity in heating and cooling applications may change somewhat because of changes in density with temperature.

But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants.

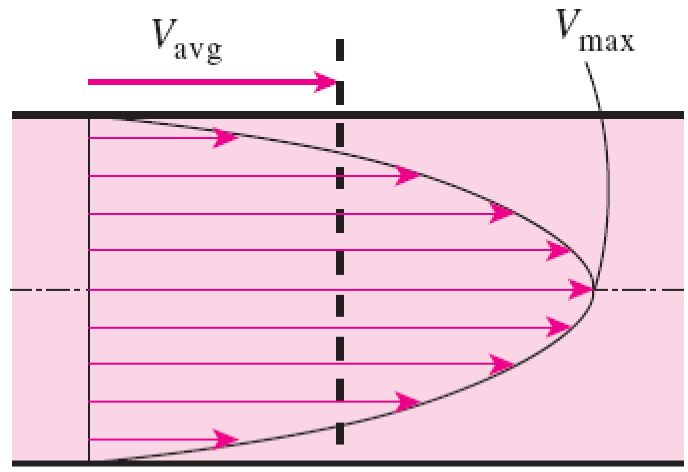


FIGURE 8–2
Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

AVERAGE VELOCITY AND TEMPERATURE

The value of the **average (mean) velocity** V_{avg} at some streamwise cross-section

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

The average velocity for incompressible flow in a circular pipe of radius R

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr$$

In fluid flow, it is convenient to work with an **average** or **mean temperature** T_m, which remains constant at a cross section. The mean temperature T_m *changes* in the flow direction whenever the fluid is heated or cooled.

$$\dot{E}_{fluid} = \dot{m} c_p T_m = \int_{\dot{m}} c_p T(r) \delta \dot{m} = \int_{A_c} \rho c_p T(r) u(r) dA_c$$

$$T_m = \frac{\int_{\dot{m}} c_p T(r) \delta \dot{m}}{\dot{m} c_p} = \frac{\int_0^R c_p T(r) \rho u(r) 2\pi r dr}{\rho V_{avg} (\pi R^2) c_p} = \frac{2}{V_{avg} R^2}\int_0^R T(r) u(r)\, r dr$$

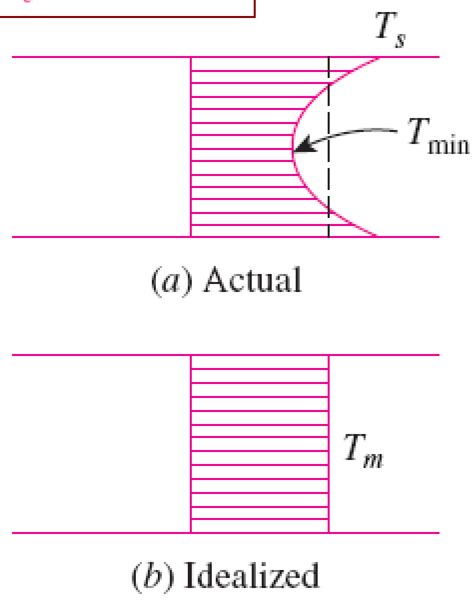


FIGURE 8–3
Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

Laminar and Turbulent Flow in Tubes

Flow in a tube can be laminar or turbulent, depending on the flow conditions.

Fluid flow is streamlined and thus laminar at low velocities, but turns turbulent as the velocity is increased beyond a critical value.

Transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some range of velocity where the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.

Most pipe flows encountered in practice are turbulent.

Laminar flow is encountered when highly viscous fluids such as oils flow in small diameter tubes or narrow passages.

Transition from laminar to turbulent flow depends on the Reynolds number as well as the degree of disturbance of the flow by *surface roughness*, *pipe vibrations*, and the *fluctuations in the flow*.

The flow in a pipe is laminar for $Re < 2300$, fully turbulent for $Re > 10,000$, and transitional in between.

Reynolds number for flow in a circular tube

$$\mathrm{Re} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg}D}{\mu} = \frac{\rho D}{\mu}\left(\frac{\dot{m}}{\rho\pi D^2/4}\right) = \frac{4\dot{m}}{\mu\pi D}$$

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number, and the friction factor are based on the **hydraulic diameter D_h**

$$D_h = \frac{4A_c}{p}$$

Circular tubes: $$D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$$

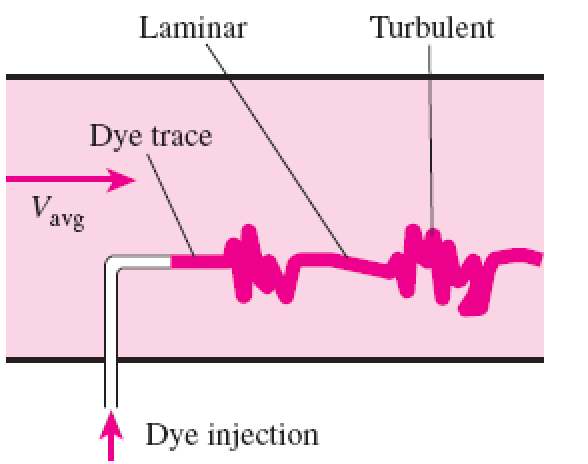


FIGURE 8–5
In the transitional flow region of the flow switches between laminar and turbulent somewhat randomly.

Under most practical conditions, the flow in a pipe is laminar for Re < 2300, fully turbulent for Re > 10,000, and transitional in between.

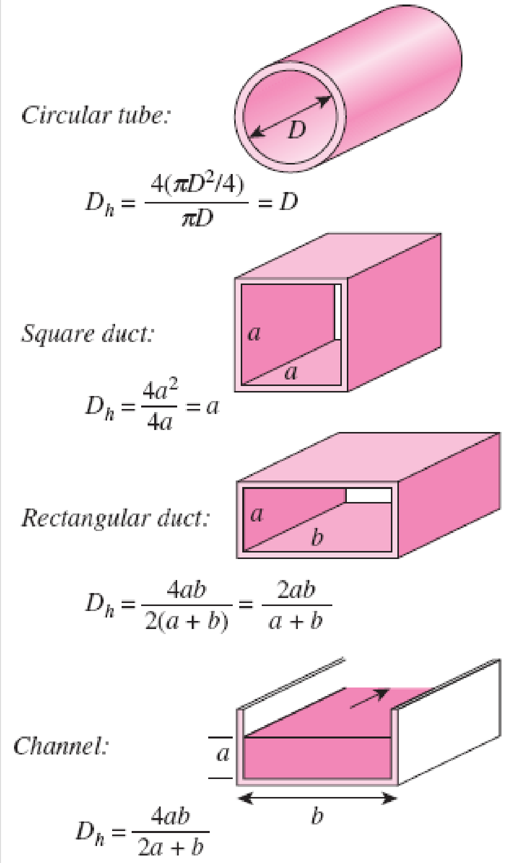


FIGURE 8–4
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes. When there is a free surface, such as in open-channel flow, the wetted perimeter includes only the walls in contact with the fluid.

THE ENTRANCE REGION

Velocity boundary layer (**boundary layer**): The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

The hypothetical boundary surface divides the flow in a pipe into two regions:

Boundary layer region: The viscous effects and the velocity changes are significant.

Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.

Hydrodynamic entrance region: The region from the pipe inlet to the point at which the velocity profile is fully developed.

Hydrodynamic entry length L_h: The length of this region.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops.

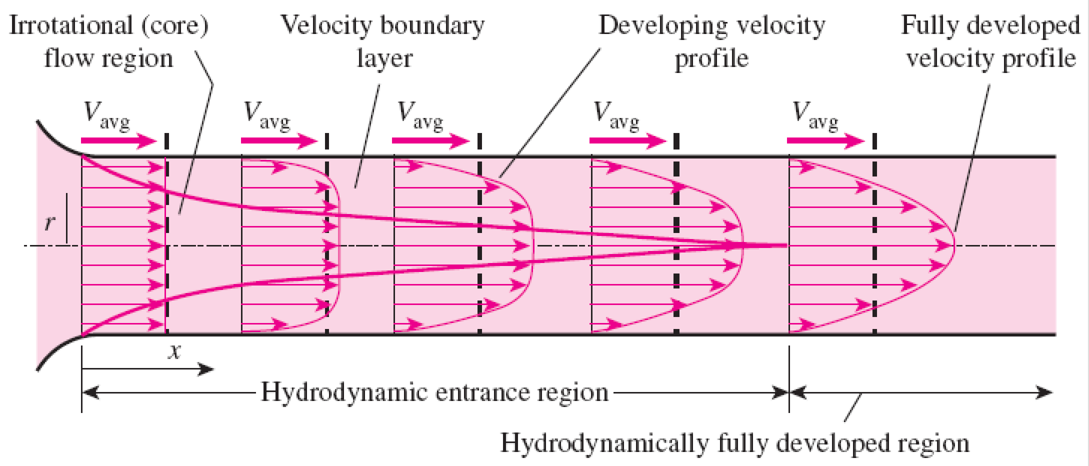


FIGURE 8–6
The development of the velocity boundary layer in a pipe. (The developed average velocity profile is parabolic in laminar flow, as shown, but much flatter or fuller in turbulent flow.)

The fluid properties in internal flow are usually evaluated at the *bulk mean fluid temperature*, which is the arithmetic average of the mean temperatures at the inlet and the exit: $T_b = (T_{m,i} + T_{m,e})/2$

Thermal entrance region: The region of flow over which the thermal boundary layer develops and reaches the tube center.

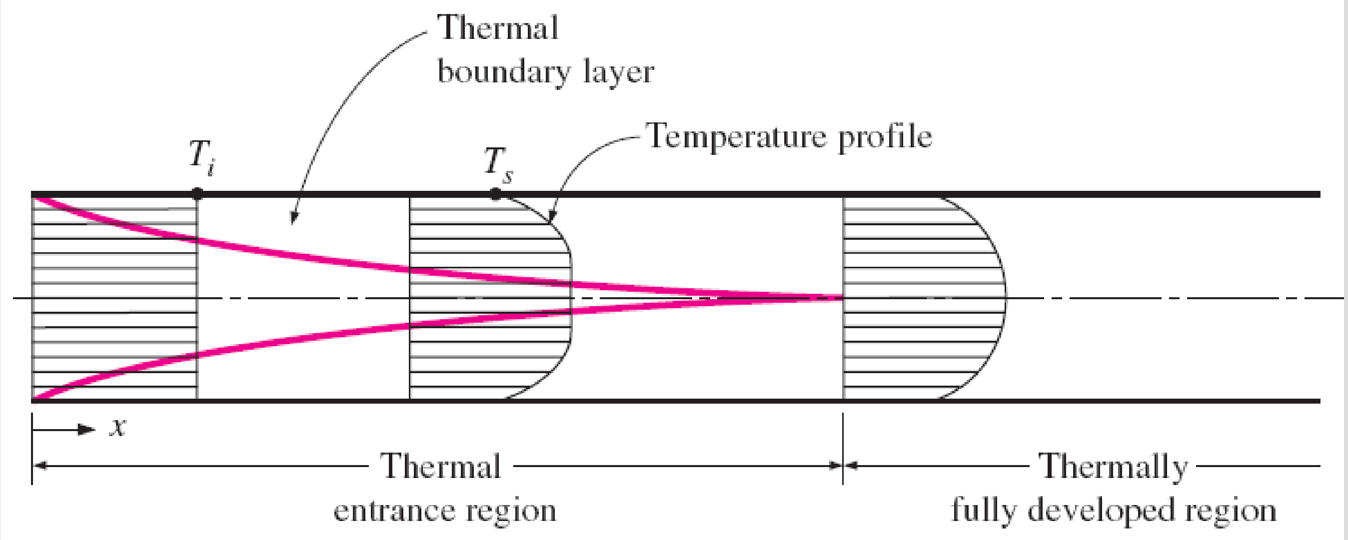
Thermal entry length: The length of this region.

Thermally developing flow: Flow in the thermal entrance region. This is the region where the temperature profile develops.

Thermally fully developed region: The region beyond the thermal entrance region in which the dimensionless temperature profile remains unchanged.

Fully developed flow: The region in which the flow is both hydrodynamically and thermally developed.

The development of the thermal boundary layer in a tube.



Hydrodynamically fully developed:

$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \longrightarrow \quad u = u(r)$$

Thermally fully developed:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$$

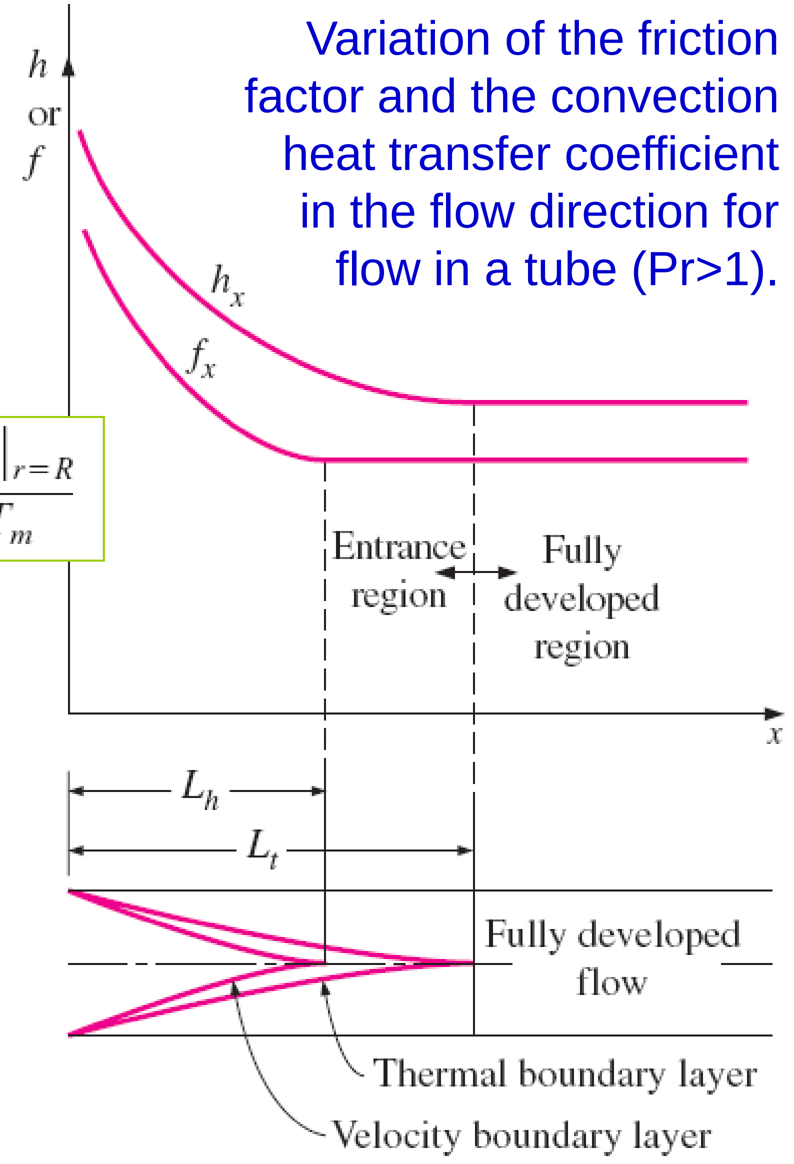
Surface heat flux

$$\dot{q}_s = h_x(T_s - T_m) = k\left.\frac{\partial T}{\partial r}\right|_{r=R} \quad \longrightarrow \quad h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m}$$

In the thermally fully developed region of a tube, the local convection coefficient is constant (does not vary with *x*).

Therefore, both *the friction* (*which is related to wall shear stress*) *and convection coefficients remain constant in the fully developed region of a tube.*

The pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *increase* the average friction factor and heat transfer coefficient for the entire tube.



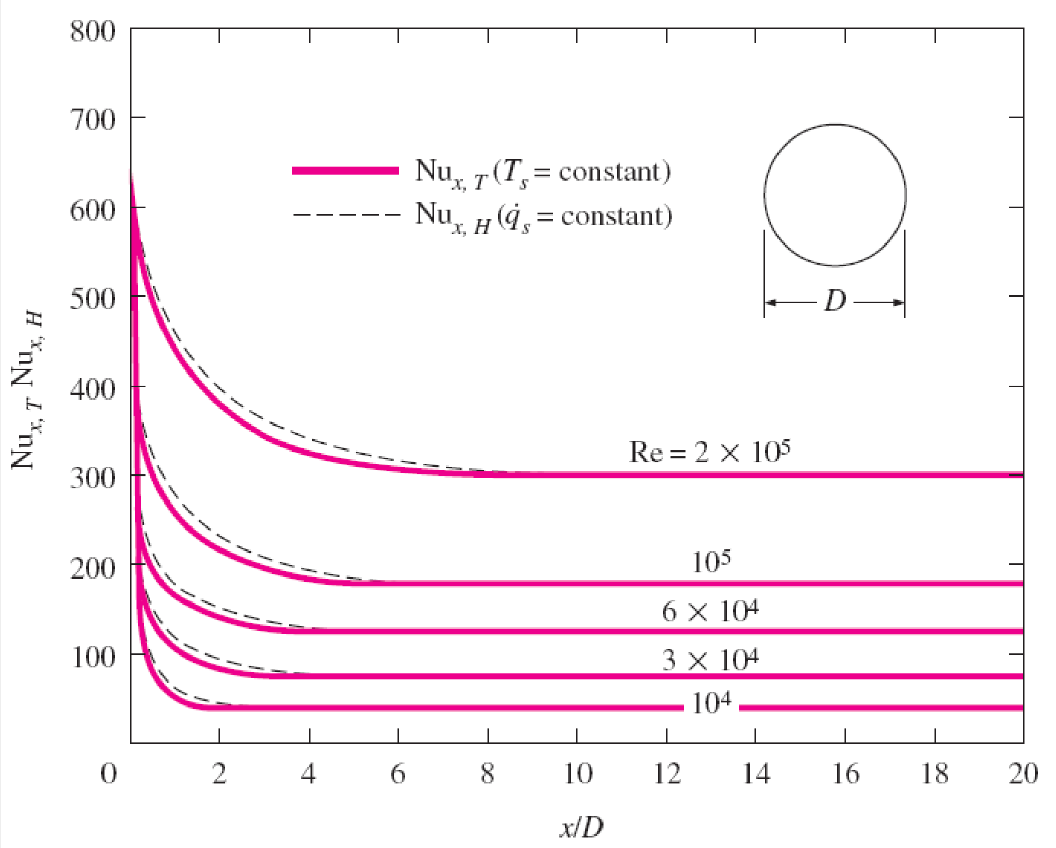
Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube (Pr>1).

Entry Lengths

$$L_{h,\,\text{laminar}} \approx 0.05\,\text{Re}\,D$$
$$L_{t,\,\text{laminar}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = \text{Pr}\,L_{h,\,\text{laminar}}$$

$$L_{h,\,\text{turbulent}} \approx L_{t,\,\text{turbulent}} \approx 10D$$

- The Nusselt numbers and thus *h* values are much higher in the entrance region.
- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for $x > 10D$.
- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions.



Variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux.

GENERAL THERMAL ANALYSIS

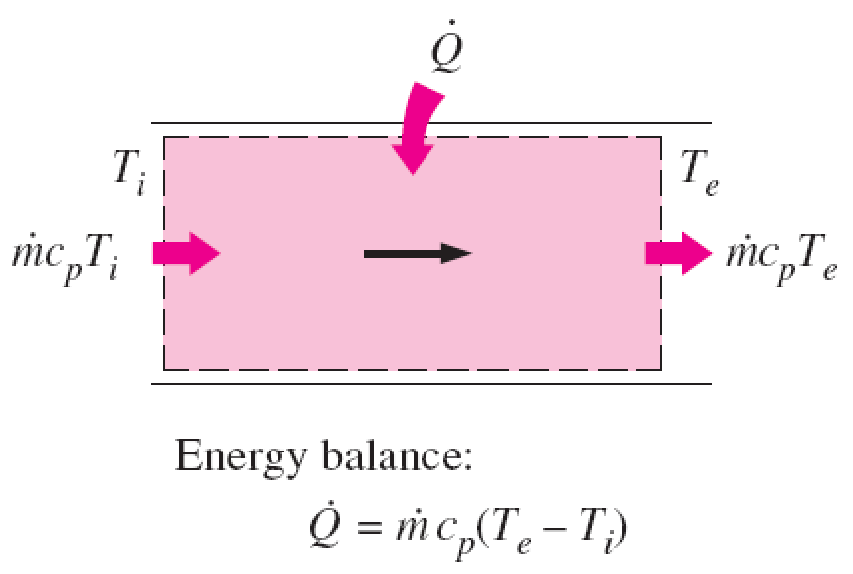
Rate of heat transfer

$$\dot{Q} = \dot{m} c_p (T_e - T_i) \qquad \text{(W)}$$

Surface heat flux

$$\dot{q}_s = h_x (T_s - T_m) \qquad (\text{W/m}^2)$$

h_x the *local* heat transfer coefficient



The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

The thermal conditions at the surface can be approximated to be

constant surface temperature (T_s= const)

constant surface heat flux (q_s = const)

The constant surface temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube.

The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

We may have either T_s = constant or q_s = constant at the surface of a tube, but not both.

Constant Surface Heat Flux (q_s = constant)

Rate of heat transfer:

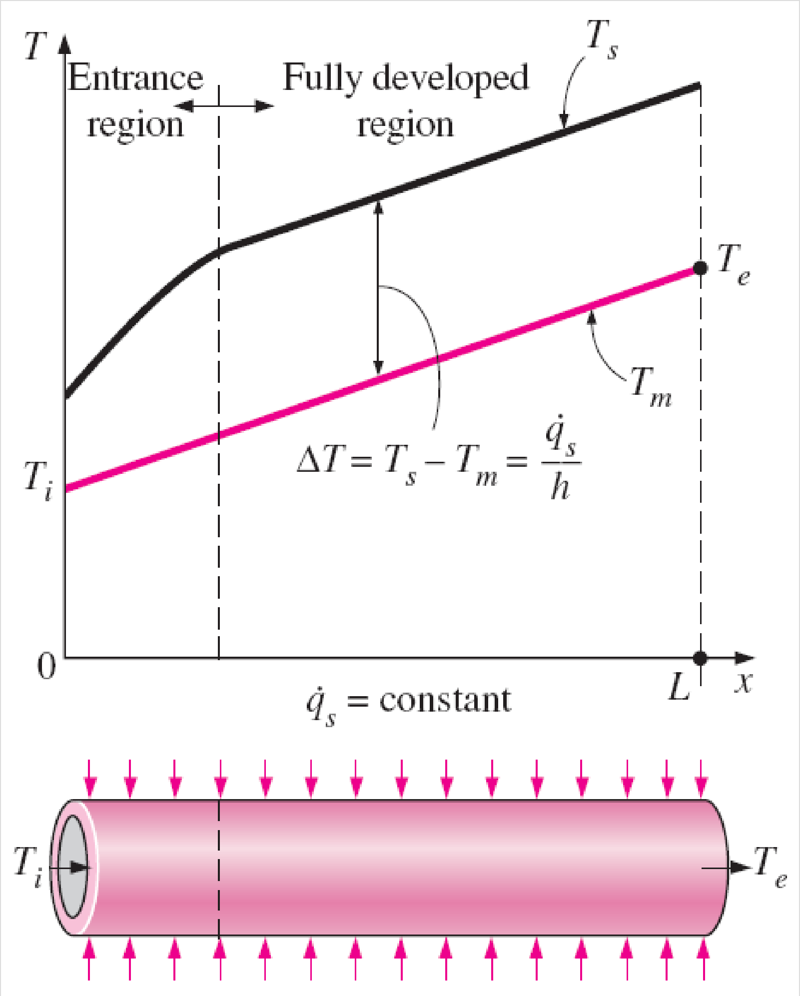
$$\dot{Q} = \dot{q}_s A_s = \dot{m} c_p (T_e - T_i) \qquad \text{(W)}$$

Mean fluid temperature at the tube exit:

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} c_p}$$

Surface temperature:

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$



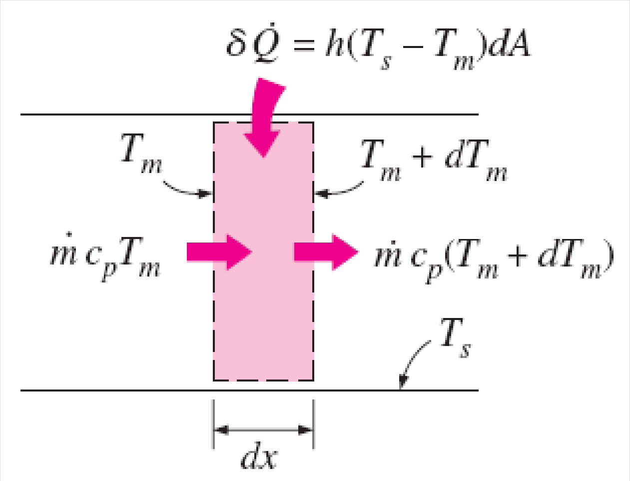
Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

$$\dot{m}c_p\,dT_m = \dot{q}_s(p\,dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant}$$

$$\frac{dT_m}{dx} = \frac{dT_s}{dx}$$

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0$$

$$\longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$

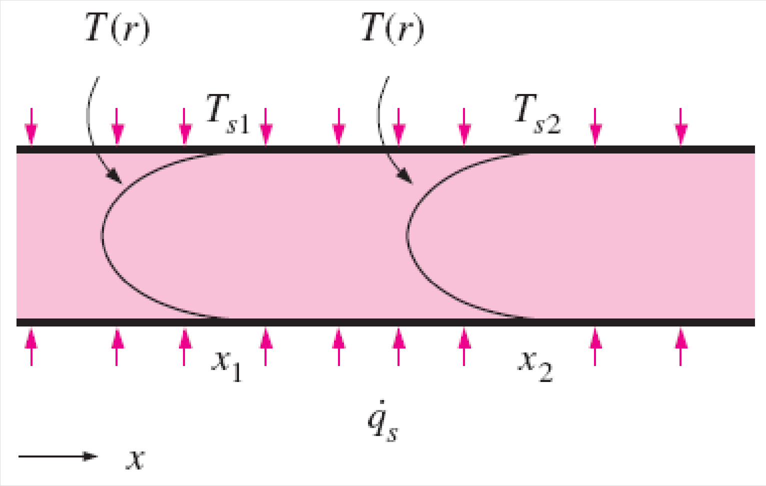


Energy interactions for a differential control volume in a tube.

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant}$$

Circular tube:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{avg} c_p R} = \text{constant}$$



The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux.

Constant Surface Temperature (T_s = constant)

Rate of heat transfer to or from a fluid flowing in a tube

$$\dot{Q} = hA_s\Delta T_{avg} = hA_s(T_s - T_m)_{avg} \qquad \text{(W)}$$

Two suitable ways of expressing ΔT_{avg}

- arithmetic mean temperature difference
- logarithmic mean temperature difference

Arithmetic mean temperature difference

$$\Delta T_{avg} \approx \Delta T_{am} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2} = T_s - T_b$$

Bulk mean fluid temperature: $T_b = (T_i + T_e)/2$

By using arithmetic mean temperature difference, we assume that the mean fluid temperature varies linearly along the tube, which is hardly ever the case when T_s = constant.

This simple approximation often gives acceptable results, but not always.

Therefore, we need a better way to evaluate ΔT_{avg}.

$$\dot{m} c_p \, dT_m = h(T_s - T_m) dA_s$$

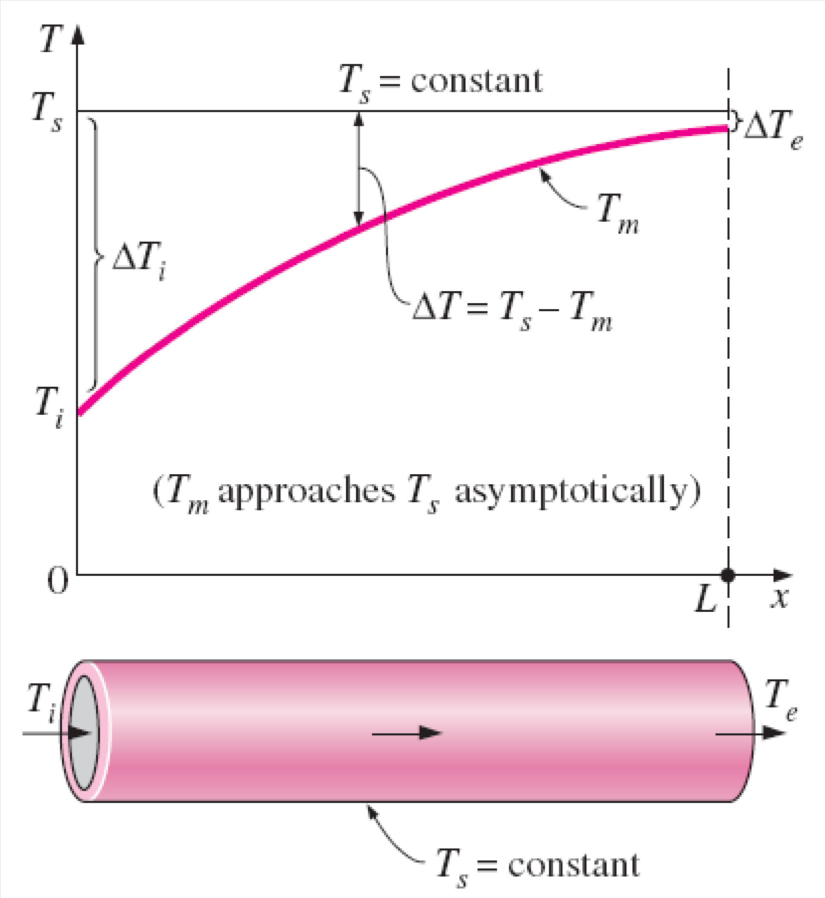
$$dA_s = p\,dx \qquad dT_m = -d(T_s - T_m)$$

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m} c_p} dx$$

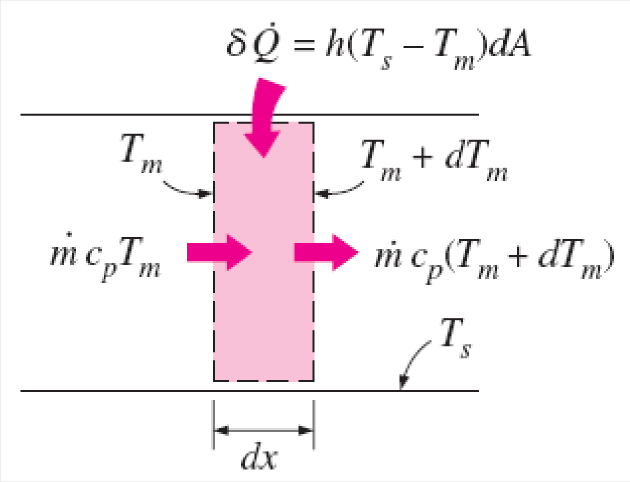
Integrating from $x = 0$ (tube inlet, $T_m = T_i$) to $x = L$ (tube exit, $T_m = T_e$)

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} c_p}$$

$$T_e = T_s - (T_s - T_i) \exp(-hA_s/\dot{m} c_p)$$



The variation of the *mean fluid* temperature along the tube for the case of constant temperature.



Energy interactions for a differential control volume in a tube.

$$\dot{Q} = hA_s\Delta T_{\ln}$$

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

log mean temperature difference

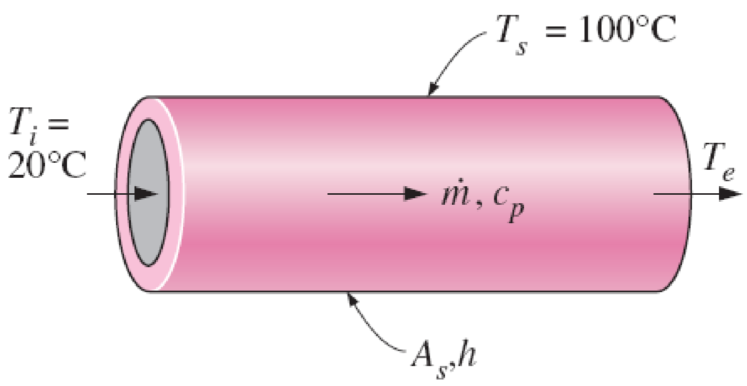
NTU: *Number of transfer units.* A measure of the effectiveness of the heat transfer systems.

For NTU = 5, $T_e = T_s$, and the limit for heat transfer is reached.

A small value of NTU indicates more opportunities for heat transfer.

$\Delta T_{\ln}$ is an *exact* representation of the *average temperature difference* between the fluid and the surface.

When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent.



NTU = $hA_s / \dot{m}c_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

EXAMPLE 8–1 **Heating of Water in a Tube by Steam**

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 $W/m^2 \cdot C$, determine the length of the tube required in order to heat the water to 115°C (Fig. 8–16).

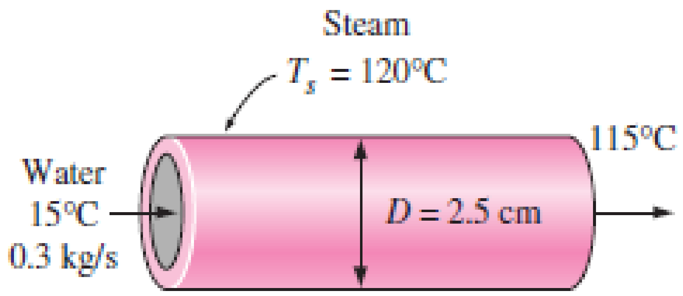


FIGURE 8–16
Schematic for Example 8–1.

SOLUTION Water is heated by steam in a circular tube. The tube length required to heat the water to a specified temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Fluid properties are constant. **3** The convection heat transfer coefficient is constant. **4** The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties The specific heat of water at the bulk mean temperature of (15 + 115)/2 = 65°C is 4187 J/kg · °C. The heat of condensation of steam at 120°C is 2203 kJ/kg (Table A-9).

Analysis Knowing the inlet and exit temperatures of water, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.3\ \text{kg/s})(4.187\ \text{kJ/kg}\cdot{}^\circ\text{C})(115^\circ\text{C} - 15^\circ\text{C}) = 125.6\ \text{kW}$$

The logarithmic mean temperature difference is

$$\Delta T_e = T_s - T_e = 120^\circ\text{C} - 115^\circ\text{C} = 5^\circ\text{C}$$

$$\Delta T_i = T_s - T_i = 120^\circ\text{C} - 15^\circ\text{C} = 105^\circ\text{C}$$

$$\Delta T_{\text{ln}} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} = \frac{5 - 105}{\ln(5/105)} = 32.85^\circ\text{C}$$

The heat transfer surface area is

$$\dot{Q} = hA_s\Delta T_{\text{ln}} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\text{ln}}} = \frac{125.6\ \text{kW}}{(0.8\ \text{kW/m}^2\cdot{}^\circ\text{C})(32.85^\circ\text{C})} = 4.78\ \text{m}^2$$

Then the required length of tube becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{4.78\ \text{m}^2}{\pi(0.025\ \text{m})} = \mathbf{61\ m}$$

LAMINAR FLOW IN TUBES

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

Taking the limit as $dr, dx \to 0$ gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

Substituting $\tau = -\mu(du/dr)$ and taking μ = constant

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

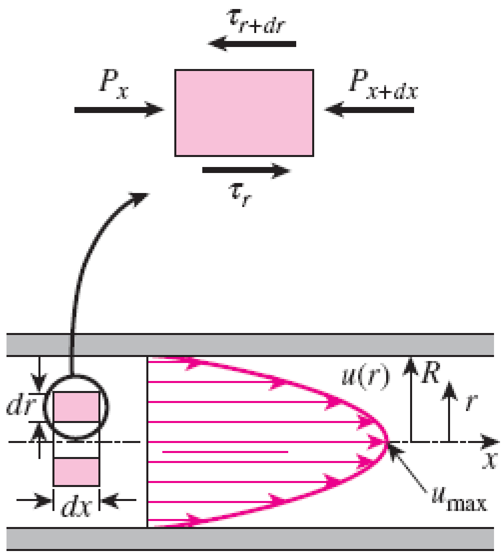
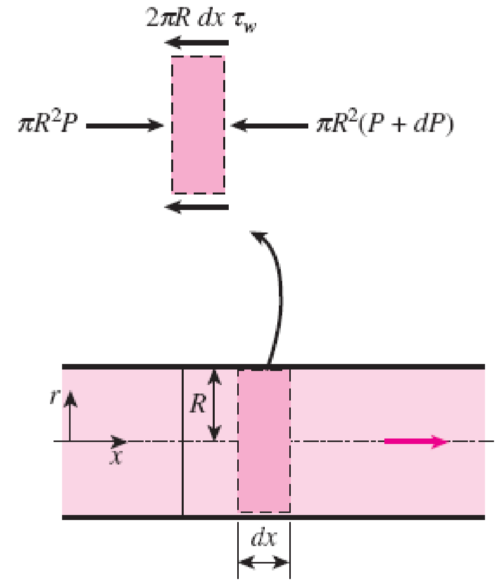


FIGURE 8–17
Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal tube in fully developed laminar flow.



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

FIGURE 8–18
Free-body diagram of a fluid disk element of radius R and length dx in fully developed laminar flow in a horizontal tube.

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

The velocity profile $u(r)$ is obtained by applying the boundary conditions $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$ (the no-slip condition at the tube wall). We get

Therefore, the velocity profile in fully developed laminar flow in a tube is *parabolic* with a maximum at the centerline and a minimum (zero) at the tube wall. Also, the axial velocity u is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

Velocity profile

The maximum velocity occurs at the centerline, $r = 0$:

$$u_{max} = 2V_{avg}$$

The average velocity in fully developed laminar pipe flow is one-half of the maximum velocity.

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow.

Pressure Drop

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

$$\text{Laminar flow:} \qquad \Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

$$\text{Pressure loss:} \qquad \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

where $\rho V^2_{avg}/2$ is the *dynamic pressure* and f is the **Darcy friction factor**,

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

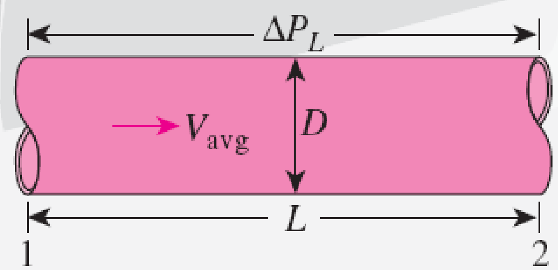
In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

$$\text{Circular tube, laminar:} \qquad f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

head loss

Pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L.



$$\text{Pressure loss: } \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

$$\text{Head loss: } h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

FIGURE 8–19
The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular tubes, and pipes with smooth or rough surfaces.

The head loss h_L represents *the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe*. The head loss is caused by viscosity, and it is directly related to the wall shear stress.

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

The required pumping power *to overcome the pressure loss:*

$$\dot{W}_{pump,\,L} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

Horizontal tube:

$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

The average velocity for laminar flow

$$\dot{V} = V_{avg}A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$

Poiseuille's law

For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the pipe.

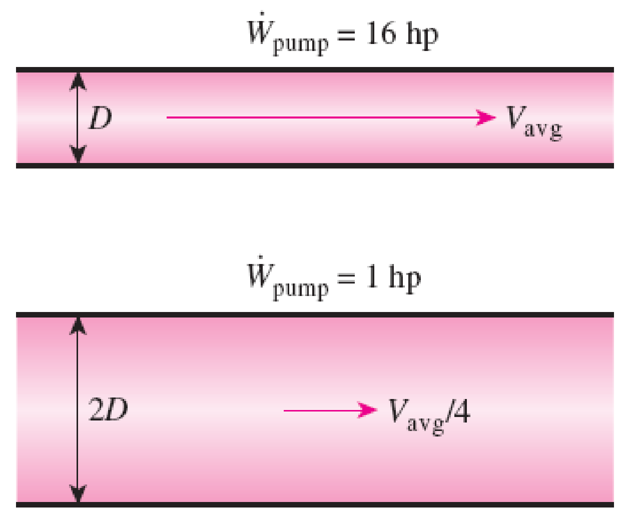


FIGURE 8–20
The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the tube diameter.

Temperature Profile and the Nusselt Number

$$\dot{m}c_pT_x - \dot{m}c_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$\dot{m} = \rho u A_c = \rho u(2\pi r dr)$$

$$\rho c_p u \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

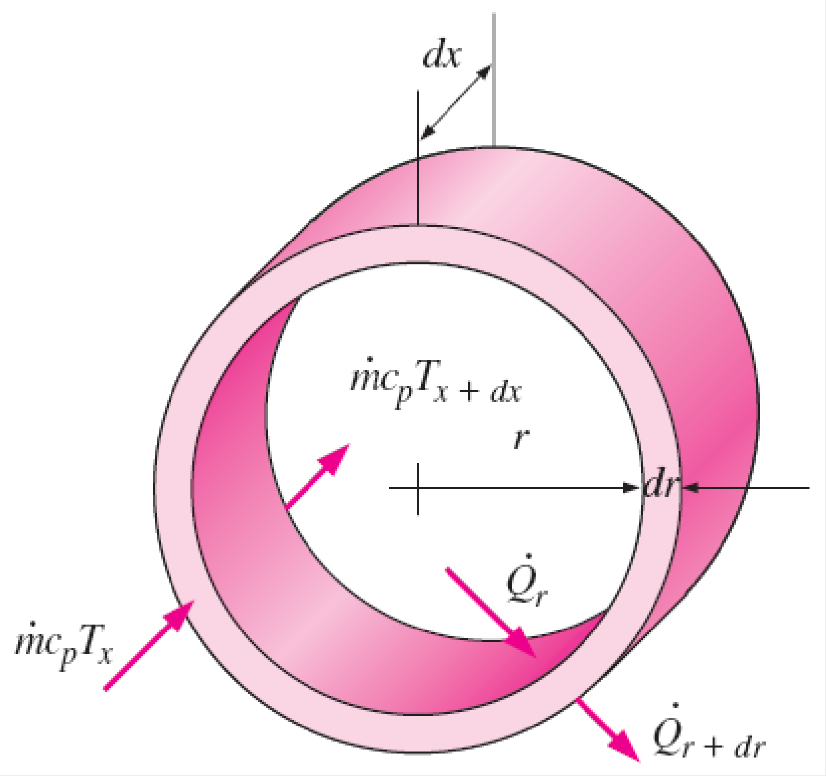
$$u\frac{\partial T}{\partial x} = -\frac{1}{2\rho c_p \pi r dx}\frac{\partial \dot{Q}}{\partial r}$$

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-k2\pi r dx\frac{\partial T}{\partial r}\right) = -2\pi k dx\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\alpha = k/\rho c_p$$

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right)$$

The rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction.



The differential volume element used in the derivation of energy balance relation.

Constant Surface Heat Flux

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{\text{avg}} c_p R} = \text{constant}$$

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

$$T = \frac{\dot{q}_s}{kR}\left(r^2 - \frac{r^4}{4R^2}\right) + C_1 r + C_2$$

Applying the boundary conditions $\partial T/\partial x = 0$ at $r = 0$ (because of symmetry) and $T = T_s$ at $r = R$

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k}$$

$$\dot{q}_s = h(T_s - T_m)$$

$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\frac{k}{D}$$

Circular tube, laminar ($\dot{q}_s$ = constant):

$$\text{Nu} = \frac{hD}{k} = 4.36$$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant.

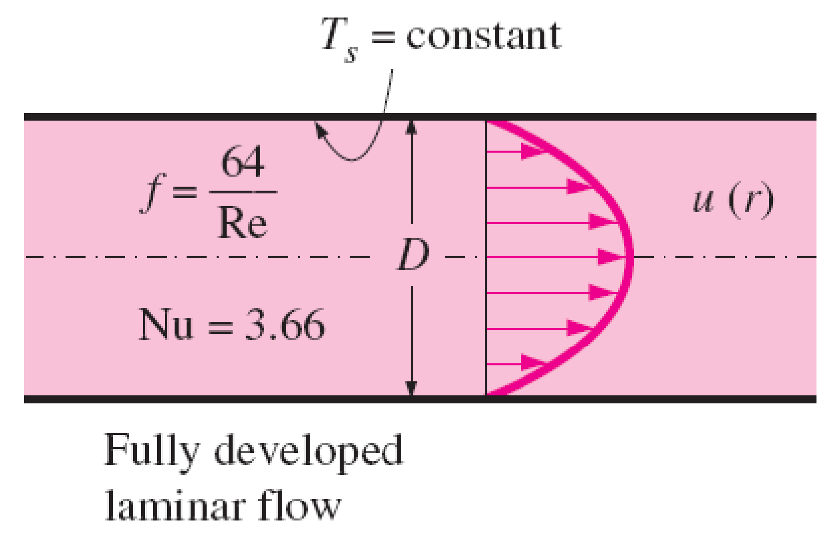
There is no dependence on the Reynolds or the Prandtl numbers.

Constant Surface Temperature

Circular tube, laminar (T_s = constant): $\quad \text{Nu} = \dfrac{hD}{k} = 3.66$

The thermal conductivity *k* for use in the Nu relations should be evaluated at the bulk mean fluid temperature.

For laminar flow, the effect of *surface roughness* on the friction factor and the heat transfer coefficient is negligible.



In laminar flow in a tube with constant surface temperature, both the *friction factor* and the *heat transfer coefficient* remain constant in the fully developed region.

Laminar Flow in Noncircular Tubes

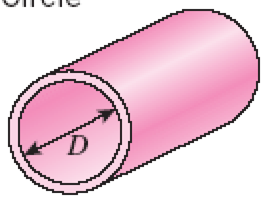
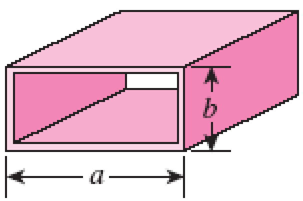
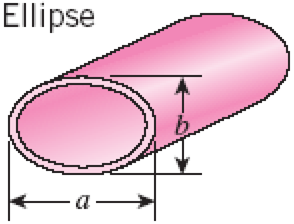
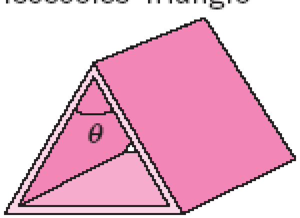
Nusselt number relations are given in Table 8-1 for *fully developed laminar flow* in tubes of various cross sections.

The Reynolds and Nusselt numbers for flow in these tubes are based on the hydraulic diameter $D_h = 4A_c/p$,

Once the Nusselt number is available, the convection heat transfer coefficient is determined from $h = k\text{Nu}/D_h$.

TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\text{Re} = V_{avg}D_h/\nu$, and $\text{Nu} = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number		Friction Factor f
		T_s = Const.	$\dot{q}_s$ = Const.	
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Developing Laminar Flow in the Entrance Region

For a circular tube of length *L* subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region*:

Entry region, laminar: $$\text{Nu} = 3.66 + \frac{0.065\,(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

The average Nusselt number is larger at the entrance region, and it approaches asymptotically to the fully developed value of 3.66 as $L \rightarrow \infty$.

When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature:

$$\text{Nu} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_s, which is evaluated at the surface temperature.

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length *L* is

Entry region, laminar: $$\text{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\text{Re}\,\text{Pr}}{1 + 0.016[(D_h/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

$$\text{Re} \leq 2800$$

FIGURE 8–23
Leo Graetz (1856–1941), a German physicist, was born at Breslau (then in Germany, now called Wroclaw and in Poland). His scientific work was first concerned with the fields of heat conduction, radiation, friction and elasticity. He was one of the first to investigate the propagation of electromagnetic energy. The dimensionless **Graetz number** describing heat transfer is named after him.

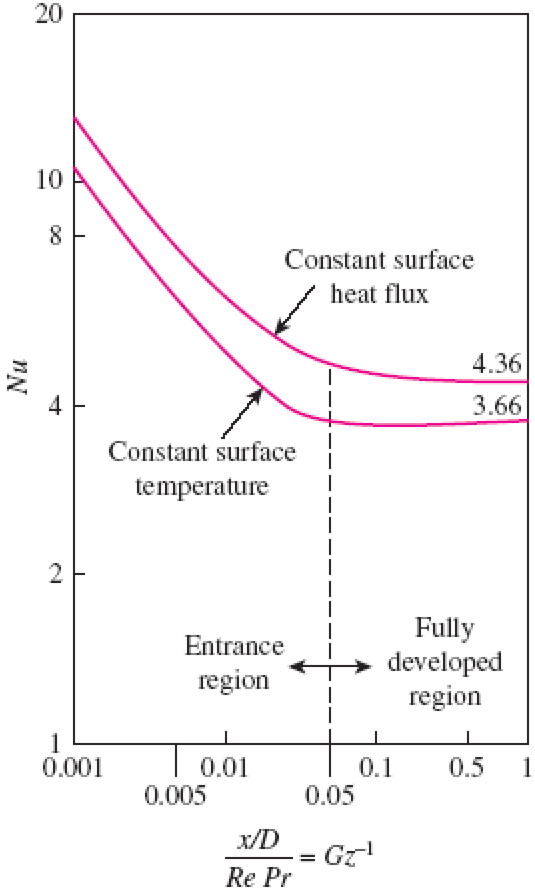


FIGURE 8–24
Local Nusselt numbers in the entry and fully developed regions for laminar flow in a circular tube for hydrodynamically developed and thermally developing flow.

EXAMPLE 8–2 **Pressure Drop in a Pipe**

Water at 40°F (ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft · h) is flowing in a 0.15-in.-diameter 30-ft-long pipe steadily at an average velocity of 3 ft/s (Fig. 8–22). Determine the pressure drop and the pumping power requirement to overcome this pressure drop.

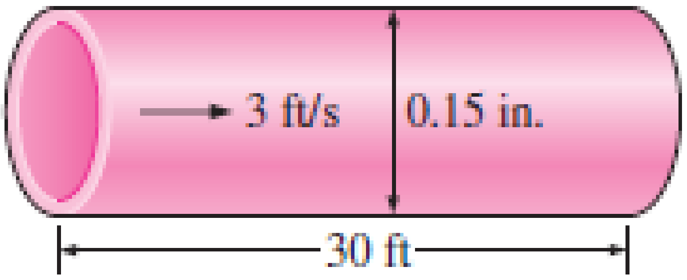


FIGURE 8–22

Schematic for Example 8–2.

SOLUTION The average flow velocity in a pipe is given. The pressure drop and the required pumping power are to be determined.

Assumptions 1 The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors.

Properties The density and dynamic viscosity of water are given to be ρ = 62.42 lbm/ft³ and μ = 3.74 lbm/ft · h = 0.00104 lbm/ft · s.

Analysis First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{(62.42\ \text{lbm/ft}^3)(3\ \text{ft/s})(0.12/12\ \text{ft})}{3.74\ \text{lbm/ft}\cdot\text{h}}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1803} = 0.0355$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} = 0.0355\,\frac{30\ \text{ft}}{0.12/12\ \text{ft}}\,\frac{(62.42\ \text{lbm/ft}^3)(3\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.174\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= \mathbf{930\ lbf/ft^2 = 6.46\ psi}$$

The volume flow rate and the pumping power requirements are

$$\dot{V} = \mathcal{V}_m A_c = \mathcal{V}_m(\pi D^2/4) = (3\ \text{ft/s})[\pi(0.12/12\ \text{ft})^2/4] = 0.000236\ \text{ft}^3/\text{s}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.000236\ \text{ft}^3/\text{s})(930\ \text{lbf/ft}^2)\left(\frac{1\ \text{W}}{0.737\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{0.30\ W}$$

Therefore, mechanical power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

EXAMPLE 8–3 **Flow of Oil in a Pipeline through a Lake**

Consider the flow of oil at 20°C in a 30-cm-diameter pipeline at an average velocity of 2 m/s (Fig. 8–23). A 200-m-long section of the pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

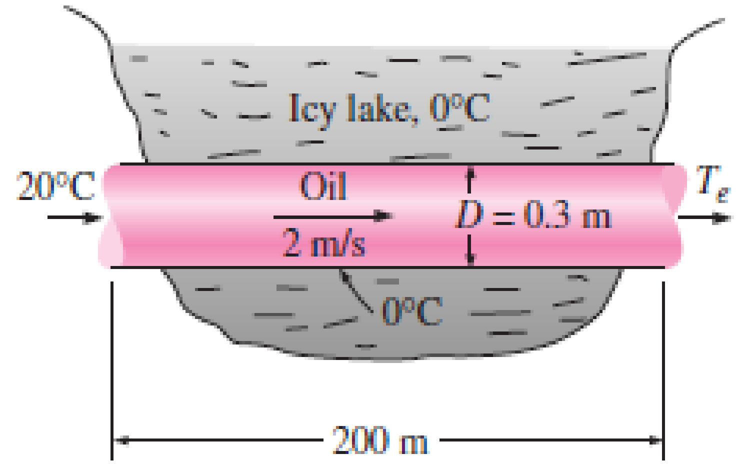


FIGURE 8–23
Schematic for Example 8–3.

SOLUTION Oil flows in a pipeline that passes through icy waters of a lake at 0°C. The exit temperature of the oil, the rate of heat loss, and the pumping power needed to overcome pressure losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties We do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is 20°C, and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evaluate the properties of the oil at the inlet temperature, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At 20°C we read (Table A-14)

$$\rho = 888 \text{ kg/m}^3 \qquad \nu = 901 \times 10^{-6} \text{ m}^2/\text{s}$$
$$k = 0.145 \text{ W/m}\cdot{}^\circ\text{C} \qquad C_p = 1880 \text{ J/kg}\cdot{}^\circ\text{C}$$
$$\text{Pr} = 10{,}400$$

Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2 \text{ m/s})(0.3 \text{ m})}{901 \times 10^{-6} \text{ m}^2/\text{s}} = 666$$

which is less than the critical Reynolds number of 2300. Therefore, the flow is laminar, and the thermal entry length in this case is roughly

$$L_t \approx 0.05 \text{ Re Pr } D = 0.05 \times 666 \times 10{,}400 \times (0.3 \text{ m}) \approx 104{,}000 \text{ m}$$

which is much greater than the total length of the pipe. This is typical of fluids with high Prandtl numbers. Therefore, we assume thermally developing flow and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 3.66 + \frac{0.065\,(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04\,[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

$$= 3.66 + \frac{0.065(0.3/200) \times 666 \times 10{,}400}{1 + 0.04[(0.3/200) \times 666 \times 10{,}400]^{2/3}}$$

$$= 37.3$$

Note that this Nusselt number is considerably higher than the fully developed value of 3.66. Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.145\ \text{W/m}}{0.3\ \text{m}}(37.3) = 18.0\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Also,

$$A_s = pL = \pi DL = \pi(0.3\ \text{m})(200\ \text{m}) = 188.5\ \text{m}^2$$

$$\dot{m} = \rho A_c \mathcal{V}_m = (888\ \text{kg/m}^3)[\tfrac{1}{4}\pi(0.3\ \text{m})^2](2\ \text{m/s}) = 125.5\ \text{kg/s}$$

Next we determine the exit temperature of oil from

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

$$= 0^\circ\text{C} - [(0-20)^\circ\text{C}]\exp\left[-\frac{(18.0\ \text{W/m}^2\cdot{}^\circ\text{C})(188.5\ \text{m}^2)}{(125.5\ \text{kg/s})(1880\ \text{J/kg}\cdot{}^\circ\text{C})}\right]$$

$$= 19.71^\circ\text{C}$$

Thus, the mean temperature of oil drops by a mere 0.29°C as it crosses the lake. This makes the bulk mean oil temperature 19.86°C, which is practically identical to the inlet temperature of 20°C. Therefore, we do not need to re-evaluate the properties.

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln \dfrac{T_s - T_e}{T_s - T_i}} = \frac{20 - 19.71}{\ln \dfrac{0 - 19.71}{0 - 20}} = -19.85°\text{C}$$

$$\dot{Q} = hA_s \Delta T_{\ln} = (18.0\ \text{W/m}^2 \cdot °\text{C})(188.5\ \text{m}^2)(-19.85°\text{C}) = \mathbf{-6.74 \times 10^4}$$

Therefore, the oil will lose heat at a rate of 67.4 kW as it flows through the pipe in the icy waters of the lake. Note that $\Delta T_{\ln}$ is identical to the arithmetic mean temperature in this case, since $\Delta T_i \approx \Delta T_e$.

(*c*) The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$f = \frac{64}{\text{Re}} = \frac{64}{666} = 0.0961$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f \frac{L}{D} \frac{\rho \mathcal{V}_m^2}{2} = 0.0961 \frac{200\ \text{m}}{0.3\ \text{m}} \frac{(888\ \text{kg/m}^3)(2\ \text{m/s})^2}{2} = 1.14 \times 10^5\ \text{N/m}^2$$

$$\dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P}{\rho} = \frac{(125.5\ \text{kg/s})(1.14 \times 10^5\ \text{N/m}^2)}{888\ \text{kg/m}^3} = \mathbf{16.1\ kW}$$

TURBULENT FLOW IN TUBES

Smooth tubes: $f = (0.790 \ln \mathrm{Re} - 1.64)^{-2}$ $\quad 3000 < \mathrm{Re} < 5 \times 10^6$

First Petukhov equation

$$\mathrm{Nu} = 0.125 f \mathrm{Re}\mathrm{Pr}^{1/3}$$

Chilton–Colburn analogy

$$f = 0.184\, \mathrm{Re}^{-0.2}$$

$$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8}\, \mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix}$$

Colburn equation

$$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8}\, \mathrm{Pr}^{n}$$

Dittus–Boelter equation

$n = 0.4$ for *heating* and 0.3 for *cooling*

When the variation in properties is large due to a large temperature difference

$$\mathrm{Nu} = 0.027\, \mathrm{Re}^{0.8} \mathrm{Pr}^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 17{,}600 \\ \mathrm{Re} \ge 10{,}000 \end{pmatrix}$$

All properties are evaluated at T_b except μ_s, which is evaluated at T_s.

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$$

Second Petukhov equation

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$$

Gnielinski relation

Liquid metals, T_s = constant: $\mathrm{Nu} = 4.8 + 0.0156\,\mathrm{Re}^{0.85}\,\mathrm{Pr}_s^{0.93}$

Liquid metals, $\dot{q}_s$ = constant: $\mathrm{Nu} = 6.3 + 0.0167\,\mathrm{Re}^{0.85}\,\mathrm{Pr}_s^{0.93}$

$(0.004 < \mathrm{Pr} < 0.01)$ $10^4 < \mathrm{Re} < 10^6$

The relations above are not very sensitive to the *thermal conditions* at the tube surfaces and can be used for both T_s = constant and q_s = constant.

Rough Surfaces

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter.

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \text{(turbulent flow)}$$

Colebrook equation

Moody chart is given in the appendix as Fig. A–20.

It presents the Darcy friction factor for pipe flow as a function of the Reynolds number and ε/D over a wide range.

$$\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

An approximate explicit relation for f was given by S. E. Haaland

In turbulent flow, wall roughness increases the heat transfer coefficient h by a factor of 2 or more. The convection heat transfer coefficient for rough tubes can be calculated approximately from *Gnielinski relation* or *Chilton–Colburn analogy* by using the friction factor determined from the *Moody chart* or the *Colebrook equation*.

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6, and are calculated from Eq. 8–74.

FIGURE 8–27
The friction factor is minimum for a smooth pipe and increases with roughness.

TABLE 8–2

Standard sizes for Schedule 40 steel pipes

Nominal Size, in	Actual Inside Diameter, in
⅛	0.269
¼	0.364
⅜	0.493
½	0.622
¾	0.824
1	1.049
1½	1.610
2	2.067
2½	2.469
3	3.068
5	5.047
10	10.02

TABLE 8–3

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ±60 percent.

Developing Turbulent Flow in the Entrance Region

The entry lengths for turbulent flow are typically short, often just 10 tube diameters long, and thus the Nusselt number determined for fully developed turbulent flow can be used approximately for the entire tube.

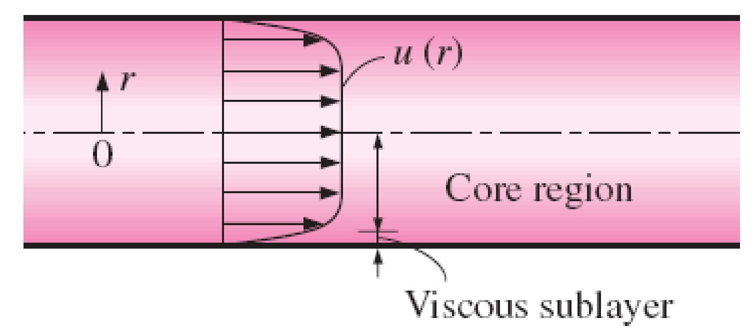
This simple approach gives reasonable results for pressure drop and heat transfer for long tubes and conservative results for short ones.

Correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature for better accuracy.

Turbulent Flow in Noncircular Tubes

Pressure drop and heat transfer characteristics of turbulent flow in tubes are dominated by the very thin viscous sublayer next to the wall surface, and the shape of the core region is not of much significance.

The turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the hydraulic diameter $D_h = 4A_c/p$.

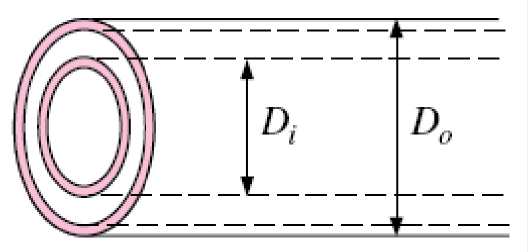


In turbulent flow, the velocity profile is nearly a straight line in the core region, and any significant velocity gradients occur in the viscous sublayer.

Flow through Tube Annulus

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i$$

The hydraulic diameter of annulus



For laminar flow, the convection coefficients for the inner and the outer surfaces are determined from

$$\text{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \text{Nu}_o = \frac{h_o D_h}{k}$$

Tube surfaces are often *roughened, corrugated,* or *finned* in order to *enhance* convection heat transfer.

For fully developed turbulent flow, h_i and h_o are approximately equal to each other, and the tube annulus can be treated as a noncircular duct with a hydraulic diameter of $D_h = D_o - D_i$.

The Nusselt number can be determined from a suitable turbulent flow relation such as the Gnielinski equation. To improve the accuracy, Nusselt number can be multiplied by the following correction factors when one of the tube walls is adiabatic and heat transfer is through the other wall:

$$F_i = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(outer wall adiabatic)}$$

$$F_o = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(inner wall adiabatic)}$$

TABLE 8–4

Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic (Kays and Perkins, 1972)

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

Heat Transfer Enhancement

Tubes with rough surfaces have much higher heat transfer coefficients than tubes with smooth surfaces.

Heat transfer in turbulent flow in a tube has been increased by as much as 400 percent by roughening the surface. Roughening the surface, of course, also increases the friction factor and thus the power requirement for the pump or the fan.

The convection heat transfer coefficient can also be increased by inducing pulsating flow by pulse generators, by inducing swirl by inserting a twisted tape into the tube, or by inducing secondary flows by coiling the tube.

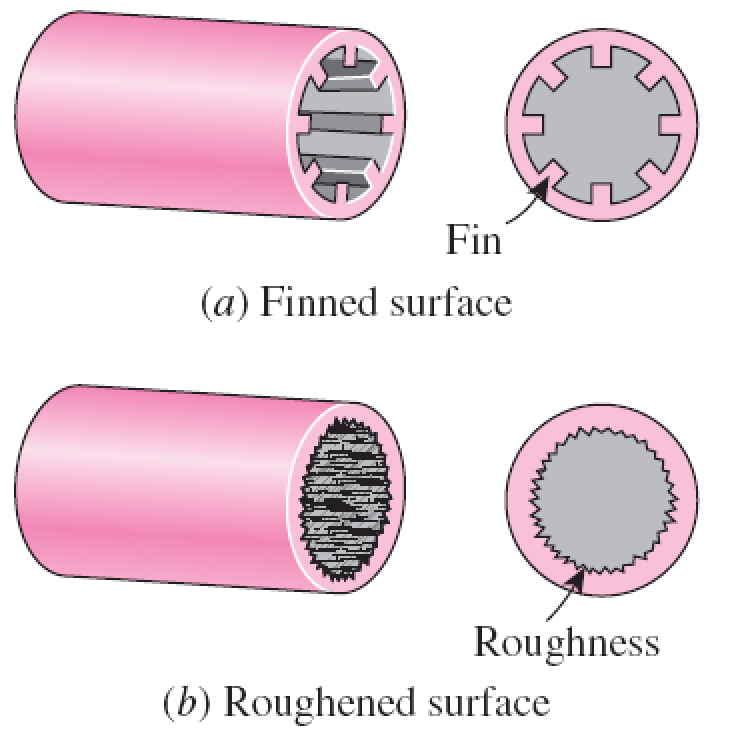


FIGURE 8–30
Tube surfaces are often *roughened*, *corrugated*, or *finned* in order to *enhance* convection heat transfer.

EXAMPLE 8–4 **Pressure Drop in a Water Pipe**

Water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h) is flowing steadily in a 2-in.-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 8–28). Determine the pressure drop and the required pumping power input for flow through a 200-ft-long section of the pipe.

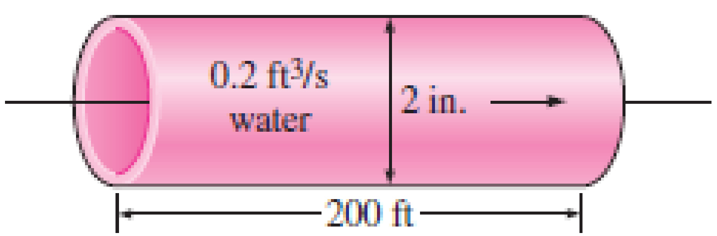


FIGURE 8–28
Schematic for Example 8–4.

SOLUTION The flow rate through a specified water pipe is given. The pressure drop and the pumping power requirements are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

Properties The density and dynamic viscosity of water are given by $\rho = 62.36$ lbm/ft^3 and $\mu = 2.713$ lbm/ft · h $= 0.0007536$ lbm/ft · s, respectively.

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2 \text{ ft}^3/\text{s}}{\pi(2/12 \text{ ft})^2/4} = 9.17 \text{ ft/s}$$

$$\text{Re} = \frac{\rho \mathcal{V} D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(9.17 \text{ ft/s})(2/12 \text{ ft})}{2.713 \text{ lbm/ft} \cdot \text{h}}\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 126{,}400$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.000007 \text{ ft}}{2/12 \text{ ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be $f = 0.0174$. Then the pressure drop and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0174\frac{200\text{ ft}}{2/12\text{ ft}}\frac{(62.36\text{ lbm/ft}^3)(9.17\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$

$$= \mathbf{1700\ lbf/ft^2 = 11.8\ psi}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.2\text{ ft}^3\text{/s})(1700\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{461\ W}$$

Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

EXAMPLE 8–5 Heating of Water by Resistance Heaters in a Tube

Water is to be heated from 15°C to 65°C as it flows through a 3-cm-internal-diameter 5-m-long tube (Fig. 8–29). The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

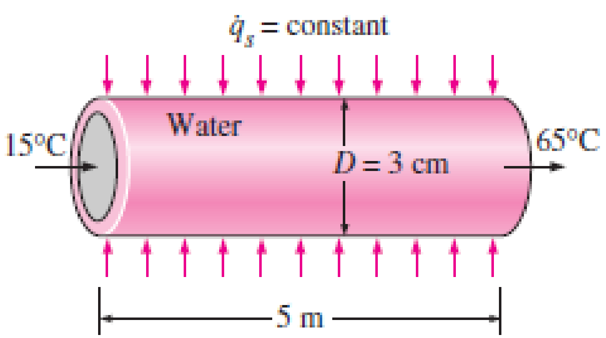


FIGURE 8–29
Schematic for Example 8–5.

SOLUTION Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (15 + 65)/2 = 40°C$ are (Table A-9).

$$\rho = 992.1 \text{ kg/m}^3 \qquad C_p = 4179 \text{ J/kg} \cdot °\text{C}$$
$$k = 0.631 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 4.32$$
$$\nu = \mu/\rho = 0.658 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis The cross sectional and heat transfer surface areas are

$$A_c = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(0.03 \text{ m})^2 = 7.069 \times 10^{-4} \text{ m}^2$$
$$A_s = pL = \pi DL = \pi(0.03 \text{ m})(5 \text{ m}) = 0.471 \text{ m}^2$$

The volume flow rate of water is given as $\dot{V} = 10$ L/min $= 0.01$ m^3/min. Then the mass flow rate becomes

$$\dot{m} = \rho\dot{V} = (992.1 \text{ kg/m}^3)(0.01 \text{ m}^3/\text{min}) = 9.921 \text{ kg/min} = 0.1654 \text{ kg/s}$$

To heat the water at this mass flow rate from 15°C to 65°C, heat must be supplied to the water at a rate of

$$\begin{aligned}\dot{Q} &= \dot{m}C_p(T_e - T_i)\\ &= (0.1654 \text{ kg/s})(4.179 \text{ kJ/kg} \cdot °\text{C})(65 - 15)°\text{C}\\ &= 34.6 \text{ kJ/s} = 34.6 \text{ kW}\end{aligned}$$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be 34.6 kW.

The surface temperature T_s of the tube at any location can be determined from

$$\dot{q}_s = h(T_s - T_m) \quad \rightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h}$$

where h is the heat transfer coefficient and T_m is the mean temperature of the fluid at that location. The surface heat flux is constant in this case, and its value can be determined from

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{34.6 \text{ kW}}{0.471 \text{ m}^2} = 73.46 \text{ kW/m}^2$$

To determine the heat transfer coefficient, we first need to find the mean velocity of water and the Reynolds number:

$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.010 \text{ m}^3/\text{min}}{7.069 \times 10^{-4} \text{ m}^2} = 14.15 \text{ m/min} = 0.236 \text{ m/s}$$

$$\text{Re} = \frac{\mathcal{V}_m D}{\nu} = \frac{(0.236 \text{ m/s})(0.03 \text{ m})}{0.658 \times 10^{-6} \text{ m}^2/\text{s}} = 10{,}760$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.03 = 0.3 \text{ m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe and determine the Nusselt number from

$$\mathrm{Nu} = \frac{hD}{k} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} = 0.023(10{,}760)^{0.8}\,(4.34)^{0.4} = 69.5$$

Then,

$$h = \frac{k}{D}\mathrm{Nu} = \frac{0.631\ \mathrm{W/m \cdot {}^\circ C}}{0.03\ \mathrm{m}}(69.5) = 1462\ \mathrm{W/m^2 \cdot {}^\circ C}$$

and the surface temperature of the pipe at the exit becomes

$$T_s = T_m + \frac{\dot{q}_s}{h} = 65^\circ\mathrm{C} + \frac{73{,}460\ \mathrm{W/m^2}}{1462\ \mathrm{W/m^2 \cdot {}^\circ C}} = 115^\circ\mathrm{C}$$

EXAMPLE 8–6 Heat Loss from the Ducts of a Heating System

Hot air at atmospheric pressure and 80°C enters an 8–m-long uninsulated square duct of cross section 0.2 m × 0.2 m that passes through the attic of a house at a rate of 0.15 m³/s (Fig. 8–30). The duct is observed to be nearly isothermal at 60°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the attic space.

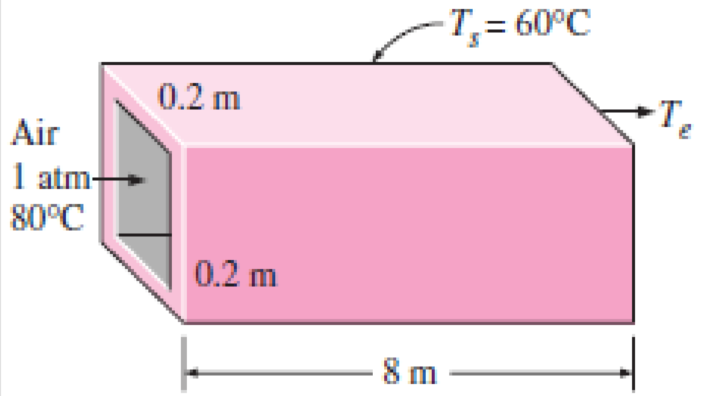


FIGURE 8–30

Schematic for Example 8–6.

SOLUTION Heat loss from uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas.

Properties We do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk mean temperature of air, which is the temperature at which the properties are to be determined. The temperature of air at the inlet is 80°C and we expect this temperature to drop somewhat as a result of heat loss through the duct whose surface is at 60°C. At 80°C and 1 atm we read (Table A-15)

$$\rho = 0.9994 \text{ kg/m}^3 \qquad C_p = 1008 \text{ J/kg}\cdot{}^\circ\text{C}$$
$$k = 0.02953 \text{ W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu = 2.097 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The characteristic length (which is the hydraulic diameter), the mean velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2 \text{ m}$$
$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.15 \text{ m}^3/\text{s}}{(0.2 \text{ m})^2} = 3.75 \text{ m/s}$$
$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(3.75 \text{ m/s})(0.2 \text{ m})}{2.097 \times 10^{-5} \text{ m}^2/\text{s}} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.2 \text{ m} = 2 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct and determine the Nusselt number from

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.3} = 0.023(35{,}765)^{0.8} (0.7154)^{0.3} = 91.4$$

Then,

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.02953 \text{ W/m} \cdot {}^\circ\text{C}}{0.2 \text{ m}}(91.4) = 13.5 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = pL = 4aL = 4 \times (0.2 \text{ m})(8 \text{ m}) = 6.4 \text{ m}^2$$

$$\dot{m} = \rho\dot{V} = (1.009 \text{ kg/m}^3)(0.15 \text{ m}^3\text{/s}) = 0.151 \text{ kg/s}$$

Next, we determine the exit temperature of air from

$$\begin{aligned} T_e &= T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \\ &= 60^\circ\text{C} - [(60 - 80)^\circ\text{C}]\exp\left[-\frac{(13.5 \text{ W/m}^2 \cdot {}^\circ\text{C})(6.4 \text{ m}^2)}{(0.151 \text{ kg/s})(1008 \text{ J/kg} \cdot {}^\circ\text{C})}\right] \\ &= \mathbf{71.3^\circ C} \end{aligned}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air become

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln\dfrac{T_s - T_e}{T_s - T_i}} = \frac{80 - 71.3}{\ln\dfrac{60 - 71.3}{60 - 80}} = -15.2^\circ\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (13.5 \text{ W/m}^2 \cdot {}^\circ\text{C})(6.4 \text{ m}^2)(-15.2^\circ\text{C}) = \mathbf{-1313 \text{ W}}$$

Summary

- Introduction
- Average Velocity and Temperature
  - ✓ Laminar and Turbulent Flow in Tubes
- The Entrance Region
  - ✓ Entry Lengths
- General Thermal Analysis
  - ✓ Constant Surface Heat Flux
  - ✓ Constant Surface Temperature
- Laminar Flow in Tubes
  - ✓ Pressure Drop
  - ✓ Temperature Profile and the Nusselt Number
  - ✓ Constant Surface Heat Flux
  - ✓ Constant Surface Temperature
  - ✓ Laminar Flow in Noncircular Tubes
  - ✓ Developing Laminar Flow in the Entrance Region
- Turbulent Flow in Tubes
  - ✓ Rough Surfaces
  - ✓ Developing Turbulent Flow in the Entrance Region
  - ✓ Turbulent Flow in Noncircular Tubes
  - ✓ Flow through Tube Annulus
  - ✓ Heat Transfer Enhancement